

PARI-GP Reference Card

(PARI-GP version 2.3.0)

Note: optional arguments are surrounded by braces {}.

Starting & Stopping GP

to enter GP, just type its name: `gp`
to exit GP, type `\q` or `quit`

Help

describe function `?function`
extended description `??keyword`
list of relevant help topics `???pattern`

Input/Output & Defaults

output previous line, the lines before `%, %', %'', etc.`
output from line n `%n`
separate multiple statements on line `;`
extend statement on additional lines `\`
extend statements on several lines `{seq1; seq2;`
comment `/* ... */`
one-line comment, rest of line ignored `\\ ...`
set default d to val `default({d},{val},flag)`
mimic behaviour of GP 1.39 `default(compatible,3)`

Metacommands

toggle timer on/off `#`
print time for last result `##`
print $%n$ in raw format `\a n`
print $%n$ in pretty format `\b n`
print defaults `\d`
set debug level to n `\g n`
set memory debug level to n `\gm n`
enable/disable logfile `\l {filename}`
print $%n$ in pretty matrix format `\m`
set output mode (raw, default, prettyprint) `\o n`
set n significant digits `\p n`
set n terms in series `\ps n`
quit GP `\q`
print the list of PARI types `\t`
print the list of user-defined functions `\u`
read file into GP `\r filename`
write $%n$ to file `\w n filename`

GP Within Emacs

to enter GP from within Emacs: `M-x gp, C-u M-x gp`
word completion `(TAB)`
help menu window `M-\c`
describe function `M-?`
display $\mathrm{T\!E\!X}$ 'd PARI manual `M-x gpman`
set prompt string `M-\p`
break line at column 100, insert `M-\\`
PARI metacommand `\letter` `M-\letter`

Reserved Variable Names

$\pi = 3.14159\dots$ `Pi`
Euler's constant $= .57721\dots$ `Euler`
square root of -1 `I`
big-oh notation `O`

PARI Types & Input Formats

`t_INT`. Integers $\pm n$
`t_REAL`. Real Numbers $\pm n.ddd$
`t_INTMOD`. Integers modulo m `Mod(n,m)`
`t_FRAC`. Rational Numbers n/m
`t_COMPLEX`. Complex Numbers $x + y * I$
`t_PADIC`. p -adic Numbers $x + O(p^k)$
`t_QUAD`. Quadratic Numbers $x + y * \text{quadgen}(D)$
`t_POLMOD`. Polynomials modulo g `Mod(f,g)`
`t_POL`. Polynomials $a * x^n + \dots + b$
`t_SER`. Power Series $f + O(x^k)$
`t_QFI/t_QFR`. Imag/Real bin. quad. forms `Qfb(a,b,c,{d})`
`t_RFRAC`. Rational Functions f/g
`t_VEC/t_COL`. Row/Column Vectors $[x,y,z], [x,y,z]~$
`t_MAT`. Matrices $[x,y;z,t;u,v]$
`t_LIST`. Lists `List([x,y,z])`
`t_STR`. Strings `"aaa"`

Standard Operators

basic operations `+, -, *, /, ^`
`i=i+1, i=i-1, i=i*j, ...` `i++, i--, i*=j,...`
euclidean quotient, remainder $x \backslash y, x \backslash y, x \% y, \text{divrem}(x,y)$
shift x left or right n bits $x << n, x >> n$ or `shift(x,n)`
comparison operators `<=, <, >=, >, ==, !=`
boolean operators (or, and, not) `||, &&, !`
sign of $x = -1, 0, 1$ `sign(x)`
maximum/minimum of x and y `max, min(x,y)`
integer or real factorial of x $x!$ or `factorial(x)`
derivative of f w.r.t. x `f'`

Conversions

Change Objects
to vector, matrix, set, list, string `Col/Vec,Mat,Set,List,Str`
create PARI object ($x \bmod y$) `Mod(x,y)`
make x a polynomial of v `Pol(x,{v})`
as above, starting with constant term `Polrev(x,{v})`
make x a power series of v `Ser(x,{v})`
PARI type of object x `type(x,{t})`
object x with precision n `prec(x,{n})`
evaluate f replacing vars by their value `eval(f)`

Select Pieces of an Object
length of x `#x` or `length(x)`
 n -th component of x `component(x,n)`
 n -th component of vector/list x `x[n]`
 (m,n) -th component of matrix x `x[m,n]`
row m or column n of matrix x `x[m,], x[,n]`
numerator of x `numerator(x)`
lowest denominator of x `denominator(x)`
Conjugates and Lifts
conjugate of a number x `conj(x)`
conjugate vector of algebraic number x `conjvec(x)`
norm of x , product with conjugate `norm(x)`
square of L^2 norm of vector x `norml2(x)`
lift of x from Mods `lift, centerlift(x)`

Random Numbers

random integer between 0 and $N - 1$ `random({N})`
get random seed `getrand()`
set random seed to s `setrand(s)`

Lists, Sets & Sorting

sort x by k th component `vecsort(x,{k},{fl=0})`
Sets (= row vector of strings with strictly increasing entries)
intersection of sets x and y `setintersect(x,y)`
set of elements in x not belonging to y `setminus(x,y)`
union of sets x and y `setunion(x,y)`
look if y belongs to the set x `setsearch(x,y,flag)`
Lists
create empty list of maximal length n `listcreate(n)`
delete all components of list l `listkill(l)`
append x to list l `listput(l,x,{i})`
insert x in list l at position i `listinsert(l,x,i)`
sort the list l `listsort(l,flag)`

Programming & User Functions

Control Statements (X : formal parameter in expression seq)
eval. seq for $a \leq X \leq b$ `for(X=a,b,seq)`
eval. seq for X dividing n `fordiv(n,X,seq)`
eval. seq for primes $a \leq X \leq b$ `forprime(X=a,b,seq)`
eval. seq for $a \leq X \leq b$ stepping s `forstep(X=a,b,s,seq)`
multivariable for `forvec(X=v,seq)`
if $a \neq 0$, evaluate seq_1 , else seq_2 `if(a,{seq1},{seq2})`
evaluate seq until $a \neq 0$ `until(a,seq)`
while $a \neq 0$, evaluate seq `while(a,seq)`
exit n innermost enclosing loops `break({n})`
start new iteration of n th enclosing loop `next({n})`
return x from current subroutine `return(x)`
error recovery (try seq_1) `trap({err},{seq2},{seq1})`
Input/Output
prettyprint args with/without newline `printp(), printp1()`
print args with/without newline `print(), print1()`
read a string from keyboard `input()`
reorder priority of variables x,y,z `reorder([x,y,z])`
output $args$ in $\mathrm{T\!E\!X}$ format `printtex(args)`
write $args$ to file `write, write1, writetex(file,args)`
read file into GP `read({file})`

Interface with User and System
allocates a new stack of s bytes `allocatemem({s})`
execute system command a `system(a)`
as above, feed result to GP `extern(a)`
install function from library `install(f,code,{gpf},{lib})`
alias old to new `alias(new,old)`
new name of function f in GP 2.0 `whatnow(f)`

User Defined Functions
`name(formal vars) = local(local vars); seq`
`struct.member = seq`
kill value of variable or function x `kill(x)`
declare global variables `global(x,...)`

Iterations, Sums & Products

numerical integration `intnum(X=a,b,expr,flag)`
sum $expr$ over divisors of n `sumdiv(n,X,expr)`
sum $X = a$ to $X = b$, initialized at x `sum(X=a,b,expr,{x})`
sum of series $expr$ `suminf(X=a,expr)`
sum of alternating/positive series `sumalt, sumpos`
product $a \leq X \leq b$, initialized at x `prod(X=a,b,expr,{x})`
product over primes $a \leq X \leq b$ `prodeuler(X=a,b,expr)`
infinite product $a \leq X \leq \infty$ `prodinf(X=a,expr)`
real root of $expr$ between a and b `solve(X=a,b,expr)`

Vectors & Matrices

dimensions of matrix x	<code>matsize(x)</code>
concatenation of x and y	<code>concat($x, \{y\}$)</code>
extract components of x	<code>vecextract($x, y, \{z\}$)</code>
transpose of vector or matrix x	<code>mattranspose(x)</code> or <code>x-</code>
adjoint of the matrix x	<code>matadjoin(x)</code>
eigenvectors of matrix x	<code>mateigen(x)</code>
characteristic polynomial of x	<code>charpoly($x, \{v\}, flag$)</code>
minimal polynomial of x	<code>minpoly($x, \{v\}$)</code>
trace of matrix x	<code>trace(x)</code>

Constructors & Special Matrices

row vec. of $expr$ eval'd at $1 \leq i \leq n$	<code>vector($n, \{i\}, \{expr\}$)</code>
col. vec. of $expr$ eval'd at $1 \leq i \leq n$	<code>vectorv($n, \{i\}, \{expr\}$)</code>
matrix $1 \leq i \leq m, 1 \leq j \leq n$	<code>matrix($m, n, \{i\}, \{j\}, \{expr\}$)</code>
diagonal matrix whose diag. is x	<code>matdiagonal(x)</code>
$n \times n$ identity matrix	<code>matid(n)</code>
Hessenberg form of square matrix x	<code>mathess(x)</code>
$n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$	<code>mathilbert(n)</code>
$n \times n$ Pascal triangle $P_{ij} = \binom{i}{j}$	<code>matpascal($n - 1$)</code>
companion matrix to polynomial x	<code>matcompanion(x)</code>

Gaussian elimination

determinant of matrix x	<code>matdet($x, flag$)</code>
kernel of matrix x	<code>matker($x, flag$)</code>
intersection of column spaces of x and y	<code>matintersect(x, y)</code>
solve $M * X = B$ (M invertible)	<code>matsolve(M, B)</code>
as solve, modulo D (col. vector)	<code>matsolvemod(M, D, B)</code>
one sol of $M * X = B$	<code>matinverseimage(M, B)</code>
basis for image of matrix x	<code>matimage(x)</code>
supplement columns of x to get basis	<code>mat supplement(x)</code>
rows, cols to extract invertible matrix	<code>matindexrank(x)</code>
rank of the matrix x	<code>matrank(x)</code>

Lattices & Quadratic Forms

upper triangular Hermite Normal Form	<code>mathnf(x)</code>
HNF of x where d is a multiple of $\det(x)$	<code>mathnfmod(x, d)</code>
elementary divisors of x	<code>matsnf(x)</code>
LLL-algorithm applied to columns of x	<code>qflll($x, flag$)</code>
like <code>qflll</code> , x is Gram matrix of lattice	<code>qflllgram($x, flag$)</code>
LLL-reduced basis for kernel of x	<code>matkerint(x)</code>
Z -lattice \longleftrightarrow Q -vector space	<code>matrixqz(x, p)</code>
signature of quad form $t^y * x * y$	<code>qfsign(x)</code>
decomp into squares of $t^y * x * y$	<code>qfgaussred(x)</code>
find up to m sols of $t^y * x * y \leq b$	<code>qfminim(x, b, m)</code>
$v, v[i] :=$ number of sols of $t^y * x * y = i$	<code>qfrep($x, B, flag$)</code>
eigenvals/eigenvecs for real symmetric x	<code>qfjacobi(x)</code>

Formal & p-adic Series

truncate power series or p -adic number	<code>truncate(x)</code>
valuation of x at p	<code>valuation(x, p)</code>
Dirichlet and Power Series	
Taylor expansion around 0 of f w.r.t. x	<code>taylor(f, x)</code>
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	<code>serconvol(x, y)</code>
$f = \sum a_k * t^k$ from $\sum (a_k / k!) * t^k$	<code>serlaplace(f)</code>
reverse power series F so $F(f(x)) = x$	<code>serreverse(f)</code>
Dirichlet series multiplication / division	<code>dirmul, dirdiv(x, y)</code>
Dirichlet Euler product (b terms)	<code>direuler($p = a, b, expr$)</code>

p-adic Functions

Teichmuller character of x	<code>teichmuller(x)</code>
Newton polygon of f for prime p	<code>newtonpoly(f, p)</code>

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Polynomials & Rational Functions

degree of f	<code>poldegree(f)</code>
coefficient of degree n of f	<code>polcoeff(f, n)</code>
round coeffs of f to nearest integer	<code>round($f, \{&e\}$)</code>
gcd of coefficients of f	<code>content(f)</code>
replace x by y in f	<code>subst(f, x, y)</code>
discriminant of polynomial f	<code>poldisc(f)</code>
resultant of f and g	<code>polresultant($f, g, flag$)</code>
as above, give $[u, v, d], xu + yv = d$	<code>bezoutres(x, y)</code>
derivative of f w.r.t. x	<code>deriv(f, x)</code>
formal integral of f w.r.t. x	<code>intformal(f, x)</code>
reciprocal poly $x^{\deg f} f(1/x)$	<code>polrecip(f)</code>
interpol. pol. eval. at a	<code>polinterpolate($X, \{Y\}, \{a\}, \{&e\}$)</code>
initialize t for Thue equation solver	<code>thueinit(f)</code>
solve Thue equation $f(x, y) = a$	<code>thue($t, a, \{sol\}$)</code>

Roots and Factorization

number of real roots of $f, a < x \leq b$	<code>polsturm($f, \{a\}, \{b\}$)</code>
complex roots of f	<code>polroots(f)</code>
symmetric powers of roots of f up to n	<code>polsym(f, n)</code>
roots of f mod p	<code>polrootsmod($f, p, flag$)</code>
factor f	<code>factor($f, \{lim\}$)</code>
factorization of f mod p	<code>factormod($f, p, flag$)</code>
factorization of f over \mathbb{F}_{p^a}	<code>factorff(f, p, a)</code>
p -adic fact. of f to prec. r	<code>factorpadic($f, p, r, flag$)</code>
p -adic roots of f to prec. r	<code>polrootspadic(f, p, r)</code>
p -adic root of f cong. to a mod p	<code>padicappr(f, a)</code>
Newton polygon of f for prime p	<code>newtonpoly(f, p)</code>

Special Polynomials

n th cyclotomic polynomial in var. v	<code>polcyclo($n, \{v\}$)</code>
d -th degree subfield of $\mathbb{Q}(\zeta_n)$	<code>polsubcyclo($n, d, \{v\}$)</code>
n -th Legendre polynomial	<code>pollegendre(n)</code>
n -th Tchebicheff polynomial	<code>poltchebi(n)</code>
Zagier's polynomial of index n, m	<code>polzagier(n, m)</code>

Transcendental Functions

real, imaginary part of x	<code>real(x), imag(x)</code>
absolute value, argument of x	<code>abs(x), arg(x)</code>
square/ n th root of x	<code>sqrtn($x, n, &z$)</code>
trig functions	<code>sin, cos, tan, cotan</code>
inverse trig functions	<code>asin, acos, atan</code>
hyperbolic functions	<code>sinh, cosh, tanh</code>
inverse hyperbolic functions	<code>asinh, acosh, atanh</code>
exponential of x	<code>exp(x)</code>
natural log of x	<code>ln(x)</code> or <code>log(x)</code>
gamma function $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$	<code>gamma(x)</code>
logarithm of gamma function	<code>lngamma(x)</code>
$\psi(x) = \Gamma'(x) / \Gamma(x)$	<code>psi(x)</code>
incomplete gamma function ($y = \Gamma(s)$)	<code>incgam($s, x, \{y\}$)</code>
exponential integral $\int_x^\infty e^{-t} / t dt$	<code>eint1(x)</code>
error function $2 / \sqrt{\pi} \int_x^\infty e^{-t^2} dt$	<code>erfc(x)</code>
dilogarithm of x	<code>dilog(x)</code>
m th polylogarithm of x	<code>polylog($m, x, flag$)</code>
U -confluent hypergeometric function	<code>hyperu(a, b, u)</code>
J -Bessel function $J_{n+1/2}(x)$	<code>besseljh(n, x)</code>
K -Bessel function of index nu	<code>besselk(nu, x)</code>

Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(x)</code>
give bit number n of integer x	<code>bittest(x, n)</code>
ceiling of x	<code>ceil(x)</code>
floor of x	<code>floor(x)</code>
fractional part of x	<code>frac(x)</code>
round x to nearest integer	<code>round($x, \{&e\}$)</code>
truncate x	<code>truncate($x, \{&e\}$)</code>
gcd/LCM of x and y	<code>gcd(x, y), lcm(x, y)</code>
gcd of entries of a vector/matrix	<code>content(x)</code>
Primes and Factorization	
add primes in v to the prime table	<code>addprimes(v)</code>
the n th prime	<code>prime(n)</code>
vector of first n primes	<code>primes(n)</code>
smallest prime $\geq x$	<code>nextprime(x)</code>
largest prime $\leq x$	<code>precprime(x)</code>
factorization of x	<code>factor($x, \{lim\}$)</code>
reconstruct x from its factorization	<code>factorback($fa, \{nf\}$)</code>

Divisors

number of distinct prime divisors	<code>omega(x)</code>
number of prime divisors with mult	<code>bigomega(x)</code>
number of divisors of x	<code>numdiv(x)</code>
row vector of divisors of x	<code>divisors(x)</code>
sum of (k -th powers of) divisors of x	<code>sigma($x, \{k\}$)</code>

Special Functions and Numbers

binomial coefficient $\binom{x}{y}$	<code>binomial(x, y)</code>
Bernoulli number B_n as real	<code>bernreal(n)</code>
Bernoulli vector B_0, B_2, \dots, B_{2n}	<code>bernvec(n)</code>
n th Fibonacci number	<code>fibonacci(n)</code>
number of partitions of n	<code>numbpart(n)</code>
Euler ϕ -function	<code>eulerphi(x)</code>
Möbius μ -function	<code>moebius(x)</code>
Hilbert symbol of x and y (at p)	<code>hilbert($x, y, \{p\}$)</code>
Kronecker-Legendre symbol $(\frac{x}{y})$	<code>kronecker(x, y)</code>

Miscellaneous

integer or real factorial of x	<code>x!</code> or <code>fact(x)</code>
integer square root of x	<code>sqrntint(x)</code>
solve $z \equiv x$ and $z \equiv y$	<code>chinese(x, y)</code>
minimal u, v so $xu + yv = \gcd(x, y)$	<code>bezout(x, y)</code>
multiplicative order of x (intmod) (i=0)	<code>znorder($x, \{o\}$)</code>
primitive root mod prime power q	<code>znprimroot(q)</code>
structure of $(\mathbb{Z}/n\mathbb{Z})^*$	<code>znstar(n)</code>
continued fraction of x	<code>contfrac($x, \{b\}, \{lmax\}$)</code>
last convergent of continued fraction x	<code>contfracpnqn(x)</code>
best rational approximation to x	<code>bestappr(x, k)</code>

True-False Tests

is x the disc. of a quadratic field?	<code>isfundamental(x)</code>
is x a prime?	<code>isprime(x)</code>
is x a strong pseudo-prime?	<code>ispseudoprime(x)</code>
is x square-free?	<code>issquarefree(x)</code>
is x a square?	<code>Z_issquare($x, \{&n\}$)</code>
is pol irreducible?	<code>polisirreducible(pol)</code>

Based on an earlier version by Joseph H. Silverman
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PARI-GP Reference Card (2)

(PARI-GP version 2.3.0)

Elliptic Curves

Elliptic curve initially given by 5-tuple $E = [a_1, a_2, a_3, a_4, a_6]$. Points are $[x, y]$, the origin is $[0]$.

Initialize elliptic struct. ell , i.e create `ellinit($E, flag$)`

$a_1, a_2, a_3, a_4, a_6, b_2, b_4, b_6, b_8, c_4, c_6, disc, j$. This data can be recovered by typing $ell.a1, \dots, ell.j$. If fl omitted, also

E defined over **R**

x -coords. of points of order 2	<code>ell.roots</code>
real and complex periods	<code>ell.omega</code>
associated quasi-periods	<code>ell.eta</code>
volume of complex lattice	<code>ell.area</code>

E defined over $\mathbf{Q}_p, |j|_p > 1$

x -coord. of unit 2 torsion point

Tate's $[u^2, u, q]$

Mestre's w

change curve E using $v = [u, r, s, t]$ `ellchangecurve(ell, v)`

change point z using $v = [u, r, s, t]$ `ellchangepoint(z, v)`

cond, min mod, Tamagawa num $[N, v, c]$ `ellglobalred(ell)`

Kodaira type of p fiber of E `elllocalred(ell, p)`

add points $z_1 + z_2$ `elladd(ell, z_1, z_2)`

subtract points $z_1 - z_2$ `ellsub(ell, z_1, z_2)`

compute $n \cdot z$ `ellpow(ell, z, n)`

check if z is on E `ellisoncurve(ell, z)`

order of torsion point z `ellorder(ell, z)`

torsion subgroup with generators `elltors(ell)`

y -coordinates of point(s) for x `ellordinate(ell, x)`

canonical bilinear form taken at z_1, z_2 `ellbil(ell, z_1, z_2)`

canonical height of z `ellheight($ell, z, flag$)`

height regulator matrix for pts in x `ellheightmatrix(ell, x)`

p th coeff a_p of L -function, p prime `ellap(ell, p)`

k th coeff a_k of L -function `ellak(ell, k)`

vector of first n a_k 's in L -function `ellan(ell, n)`

$L(E, s)$, set $A \approx 1$ `elllseries($ell, s, \{A\}$)`

root number for $L(E, \cdot)$ at p `ellrootno($ell, \{p\}$)`

modular parametrization of E `elltaniyama(ell)`

point $[\wp(z), \wp'(z)]$ corresp. to z `ellztopoint(ell, z)`

complex z such that $p = [\wp(z), \wp'(z)]$ `ellpointtoz(ell, p)`

Elliptic & Modular Functions

arithmetic-geometric mean `agm(x, y)`

elliptic j -function $1/q + 744 + \dots$ `ellj(x)`

Weierstrass σ function `ellsigma($ell, z, flag$)`

Weierstrass \wp function `ellwp($ell, \{z\}, flag$)`

Weierstrass ζ function `ellzeta(ell, z)`

modified Dedekind η func. $\prod(1 - q^n)$ `eta($x, flag$)`

Jacobi sine theta function `theta(q, z)`

k-th derivative at $z=0$ of `thetanullk(q, k)`

Weber's f functions `weber($x, flag$)`

Riemann's zeta $\zeta(s) = \sum n^{-s}$ `zeta(s)`

Graphic Functions

crude graph of $expr$ between a and b `plot($X = a, b, expr$)`

High-resolution plot (immediate plot)

plot $expr$ between a and b `plotoh($X = a, b, expr, flag, \{n\}$)`

plot points given by lists lx, ly `plotdraw($lx, ly, flag$)`

terminal dimensions `plotsizes()`

Rectwindow functions

init window w , with size x, y `plotinit(w, x, y)`

erase window w `plotkill(w)`

copy w to w_2 with offset (dx, dy) `plotcopy(w, w_2, dx, dy)`

scale coordinates in w `plotscale(w, x_1, x_2, y_1, y_2)`

`plotoh` in w `plotrecth($w, X = a, b, expr, flag, \{n\}$)`

`plotdraw` in w `plotrecthdraw($w, data, flag$)`

draw window w_1 at $(x_1, y_1), \dots$ `plotdraw($[[w_1, x_1, y_1], \dots]$)`

Low-level Rectwindow Functions

set current drawing color in w to c `plotcolor(w, c)`

current position of cursor in w `plotcursor(w)`

write s at cursor's position `plotstring(w, s)`

move cursor to (x, y) `plotmove(w, x, y)`

move cursor to $(x + dx, y + dy)$ `plotrmove(w, dx, dy)`

draw a box to (x_2, y_2) `plotbox(w, x_2, y_2)`

draw a box to $(x + dx, y + dy)$ `plotrbox(w, dx, dy)`

draw polygon `plotlines($w, lx, ly, flag$)`

draw points `plotpoints(w, lx, ly)`

draw line to $(x + dx, y + dy)$ `plotrline(w, dx, dy)`

draw point $(x + dx, y + dy)$ `plotrpoint(w, dx, dy)`

Postscript Functions

as `plotoh` `psplotoh($X = a, b, expr, flag, \{n\}$)`

as `plotdraw` `psplotdraw($lx, ly, flag$)`

as `plotdraw` `psdraw($[[w_1, x_1, y_1], \dots]$)`

Binary Quadratic Forms

create $ax^2 + bxy + cy^2$ (distance d) `qfb($a, b, c, \{d\}$)`

reduce x ($s = \sqrt{D}$, $l = \lfloor s \rfloor$) `qfbred($x, flag, \{D\}, \{l\}, \{s\}$)`

composition of forms $x*y$ or `qfbnucomp(x, y, l)`

n -th power of form x^n or `qfbnupow(x, n)`

composition without reduction `qfbcompraw(x, y)`

n -th power without reduction `qfbpowraw(x, n)`

prime form of disc. x above prime p `qfbprimeform(x, p)`

class number of disc. x `qfbclassno(x)`

Hurwitz class number of disc. x `qfbhclassno(x)`

Quadratic Fields

quadratic number $\omega = \sqrt{x}$ or $(1 + \sqrt{x})/2$ `quadgen(x)`

minimal polynomial of ω `quadpoly(x)`

discriminant of $\mathbf{Q}(\sqrt{D})$ `quaddisc(x)`

regulator of real quadratic field `quadregulator(x)`

fundamental unit in real $\mathbf{Q}(x)$ `quadunit(x)`

class group of $\mathbf{Q}(\sqrt{D})$ `quadclassunit($D, flag, \{t\}$)`

Hilbert class field of $\mathbf{Q}(\sqrt{D})$ `quadhilbert($D, flag$)`

ray class field modulo f of $\mathbf{Q}(\sqrt{D})$ `quadrday($D, f, flag$)`

General Number Fields: Initializations

A number field K is given by a monic irreducible $f \in \mathbf{Z}[X]$.

init number field structure nf `nfinit($f, flag$)`

nf members:

polynomial defining nf , $f(\theta) = 0$ `nf.pol`

number of real/complex places `nf.r1, nf.r2`

discriminant of nf `nf.disc`

T_2 matrix `nf.t2`

vector of roots of f `nf.roots`

integral basis of \mathbf{Z}_K as powers of θ `nf.zk`

different `nf.diff`

codifferent `nf.codiff`

recompute nf using current precision `nfnewprec(nf)`

init relative rmf given by $g = 0$ over K `rmfinit(nf, g)`

init bnf structure `bnfinit($f, flag$)`

bnf members: same as nf , plus

underlying nf `bnf.nf`

classgroup `bnf.clgp`

regulator `bnf.reg`

fundamental units `bnf.fu`

torsion units `bnf.tu`

$[tu, fu]$ `bnf.tufu`

compute a bnf from small bnf `bnfmake($sbnf$)`

add S -class group and units, yield bnf s `bnfsunit(nf, S)`

init class field structure bnr `bnrinit($bnf, m, flag$)`

bnr members: same as bnf , plus

underlying bnf `bnr.bnf`

structure of $(\mathbf{Z}_K/m)^*$ `bnr.zkst`

Simple Arithmetic Invariants (nf)

Elements are rational numbers, polynomials, polmods, or column vectors (on integral basis $nf.zk$).

integral basis of field def. by $f = 0$ **nfbasis**(f)
field discriminant of field $f = 0$ **nfdisc**(f)
reverse polmod $a = A(X) \bmod T(X)$ **modreverse**(a)
Galois group of field $f = 0$, $\deg f \leq 11$ **polgalois**(f)
smallest poly defining $f = 0$ **polredabs**($f, flag$)
small polys defining subfields of $f = 0$ **polred**($f, flag, \{p\}$)
small polys defining suborders of $f = 0$ **polredord**(f)
poly of degree $\leq k$ with root $x \in \mathbf{C}$ **algdep**(x, k)
small linear rel. on coords of vector x **linddep**(x)
are fields $f = 0$ and $g = 0$ isomorphic? **nfisism**(f, g)
is field $f = 0$ a subfield of $g = 0$? **nfisincl**(f, g)
compositum of $f = 0$, $g = 0$ **polcompositum**($f, g, flag$)
basic element operations (prefix **nfelt**):

(**nfelt**)**mul**, **pow**, **div**, **diveuc**, **mod**, **divrem**, **val**
express x on integer basis **nfalgtobasis**(nf, x)
express element x as a polmod **nfbasistoalg**(nf, x)
quadratic Hilbert symbol (at p) **nfhilbert**($nf, a, b, \{p\}$)
roots of g belonging to nf **nfroots**($\{nf\}, g$)
factor g in nf **nfactor**(nf, g)
factor g mod prime pr in nf **nfactormod**(nf, g, pr)
number of roots of unity in nf **nfrootsof1**(nf)
conjugates of a root θ of nf **nfgaloisconj**($nf, flag$)
apply Galois automorphism s to x **nfgaloisapply**(nf, s, x)
subfields (of degree d) of nf **nfsubfields**($nf, \{d\}$)

Dedekind Zeta Function ζ_K

ζ_K as Dirichlet series, $N(I) < b$ **dirzetak**(nf, b)
init nfz for field $f = 0$ **zetakinit**(f)
compute $\zeta_K(s)$ **zetak**($nfz, s, flag$)
Artin root number of K **bnrrootnumber**($bnr, chi, flag$)

Class Groups & Units (bnf, bnr)

$a_1, \{a_2\}, \{a_3\}$ usually $bnr, subgp$ or $bnf, module, \{subgp\}$
remove GRH assumption from bnf **bnfcertify**(bnf)
expo. of ideal x on class gp **bnfisprincipal**($bnf, x, flag$)
expo. of ideal x on ray class gp **bnrisprincipal**($bnr, x, flag$)
expo. of x on fund. units **bnfisunit**(bnf, x)
as above for S -units **bnfissunit**($bnfs, x$)
fundamental units of bnf **bnfunit**(bnf)
signs of real embeddings of $bnf.fu$ **bnfsignunit**(bnf)

Class Field Theory

ray class group structure for mod. m **bnrclass**($bnf, m, flag$)
ray class number for mod. m **bnrclassno**(bnf, m)
discriminant of class field ext **bnrdisc**($a_1, \{a_2\}, \{a_3\}$)
ray class numbers, l list of mods **bnrclassnolist**(bnf, l)
discriminants of class fields **bnrdisclist**($bnf, l, \{arch\}, flag$)
decode output from **bnrdisc** **bnfdecodemodule**(nf, fa)
is modulus the conductor? **bnrisconductor**($a_1, \{a_2\}, \{a_3\}$)
conductor of character chi **bnrconductorofchar**(bnr, chi)
conductor of extension **bnrconductor**($a_1, \{a_2\}, \{a_3\}, flag$)
conductor of extension def. by g **rnfconductor**(bnf, g)
Artin group of ext. def'd by g **rnfnormgroup**(bnr, g)
subgroups of bnr , index $\leq b$ **subgrouplist**($bnr, b, flag$)
rel. eq. for class field def'd by sub **rnfkummer**($bnr, sub, \{d\}$)
same, using Stark units (real field) **bnrstark**($bnr, sub, flag$)

PARI-GP Reference Card (2)

(PARI-GP version 2.3.0)

Ideals

Ideals are elements, primes, or matrix of generators in HNF.
is id an ideal in nf ? **nfisideal**(nf, id)
is x principal in bnf ? **bnfisprincipal**(bnf, x)
principal ideal generated by x **idealprincipal**(nf, x)
principal idele generated by x **ideleprincipal**(nf, x)
give $[a, b]$, s.t. $a\mathbf{Z}_K + b\mathbf{Z}_K = x$ **idealtwoelt**($nf, x, \{a\}$)
put ideal a ($a\mathbf{Z}_K + b\mathbf{Z}_K$) in HNF form **idealhnf**($nf, a, \{b\}$)
norm of ideal x **idealnrm**(nf, x)
minimum of ideal x (direction v) **idealmin**(nf, x, v)
LLL-reduce the ideal x (direction v) **idealred**($nf, x, \{v\}$)

Ideal Operations

add ideals x and y **idealadd**(nf, x, y)
multiply ideals x and y **idealmul**($nf, x, y, flag$)
intersection of ideals x and y **idealintersect**($nf, x, y, flag$)
 n -th power of ideal x **idealpow**($nf, x, n, flag$)
inverse of ideal x **idealin**(nf, x)
divide ideal x by y **idealdiv**($nf, x, y, flag$)
Find $(a, b) \in x \times y$, $a + b = 1$ **idealaddtoone**($nf, x, \{y\}$)

Primes and Multiplicative Structure

factor ideal x in nf **idealfactor**(nf, x)
recover x from its factorization in nf **factorback**(x, nf)
decomposition of prime p in nf **idealprimedec**(nf, p)
valuation of x at prime ideal pr **idealval**(nf, x, pr)
weak approximation theorem in nf **idealchinese**(nf, x, y)
give bid = structure of $(\mathbf{Z}_K/id)^*$ **idealstar**($nf, id, flag$)
discrete log of x in $(\mathbf{Z}_K/bid)^*$ **ideallog**(nf, x, bid)
idealstar of all ideals of norm $\leq b$ **ideallist**($nf, b, flag$)
add archimedean places **ideallistarch**($nf, b, \{ar\}, flag$)
init **prmod** structure **nfmodprinit**(nf, pr)
kernel of matrix M in $(\mathbf{Z}_K/pr)^*$ **nfkermodpr**($nf, M, prmod$)
solve $Mx = B$ in $(\mathbf{Z}_K/pr)^*$ **nfsolvemodpr**($nf, M, B, prmod$)

Galois theory over q

initializes a Galois group structure **galoisinit**($pol, \{den\}$)
action of p in **nfgaloisconj** form **galoispermopol**($G, \{p\}$)
identifies as abstract group **galoisidentify**(G)
exports a group for GAP or MAGMA **galoisexport**($G, flag$)
subgroups of the Galois group G **galoissubgroups**(G)
subfields from subgroups of G **galoissubfields**($G, flag, \{v\}$)
fixed field **galoisfixedfield**($G, perm, flag, \{v\}$)
is G abelian? **galoisisabelian**($G, flag$)
abelian number fields **galoissubcyclo**($N, H, flag, \{v\}$)

Relative Number Fields (rnf)

Extension L/K is defined by $g \in K[x]$. We have $order \subset L$.
absolute equation of L **rnfequation**($nf, g, flag$)
relative **nfalgtobasis** **rnfalgtobasis**(rnf, x)
relative **nfbasistoalg** **rnfbasistoalg**(rnf, x)
relative **idealhnf** **rnfidealhnf**(rnf, x)
relative **idealmul** **rnfidealmul**(rnf, x, y)
relative **idealtwoelt** **rnfidealtwoelt**(rnf, x)

Lifts and Push-downs

absolute \rightarrow relative repres. for x **rnfeltabstorel**(rnf, x)
relative \rightarrow absolute repres. for x **rnfeltreltoabs**(rnf, x)
lift x to the relative field **rnfeltup**(rnf, x)
push x down to the base field **rnfeltdown**(rnf, x)
idem for x ideal: (**rnfideal**)**reltoabs**, **abstorel**, **up**, **down**

Projective \mathbf{Z}_K -modules, maximal order

relative **polred** **rnfpolred**(nf, g)
relative **polredabs** **rnfpolredabs**(nf, g)
characteristic poly. of $a \bmod g$ **rnfcharpoly**($nf, g, a, \{v\}$)
relative Dedekind criterion, prime pr **rnfdedekind**(nf, g, pr)
discriminant of relative extension **rnfdisc**(nf, g)
pseudo-basis of \mathbf{Z}_L **rnfpseudobasis**(nf, g)
relative HNF basis of $order$ **rnfhnfbasis**($bnf, order$)
reduced basis for $order$ **rnflllgram**($nf, g, order$)
determinant of pseudo-matrix A **rnfdet**(nf, A)
Steinitz class of $order$ **rnfsteynitz**($nf, order$)
is $order$ a free \mathbf{Z}_K -module? **rnfisfree**($bnf, order$)
true basis of $order$, if it is free **rnfbasis**($bnf, order$)

Norms

absolute norm of ideal x **rnfidealnrmabs**(rnf, x)
relative norm of ideal x **rnfidealnrmrel**(rnf, x)
solutions of $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$ **bnfisintnorm**(bnf, x)
is $x \in \mathbf{Q}$ a norm from K ? **bnfisnorm**($bnf, x, flag$)
initialize T for norm eq. solver **rnfisnorminit**($K, pol, flag$)
is $a \in K$ a norm from L ? **rnfisnorm**($T, a, flag$)

Based on an earlier version by Joseph H. Silverman

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